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Certain Derivation on Lorentzian α - Sasakian Manifolds

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CERTAIN DERIVATION ON LORENTZIAN -SASAKIANMANIFOLDS

Strictly as per the compliance and regulations of :

Certain Derivation on Lorentzian α-Sasakian Manifolds

S.Yadav [α] & D.L.Suthar [σ]

Abstract - We classify Lorentzian α - Sasakian manifolds, which satisfy the derivation $Z(\zeta, X) \cdot Z = 0$, $Z(\zeta, X) \cdot R = 0$, $R(\zeta, X) \cdot Z = 0$, $Z(\zeta, X) \cdot S = 0$, and $Z(\zeta, X) \cdot C = 0$.

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I. Introduction

In [11], S.Tanno classified connected almost contact metric manifolds whose automorphism group possesses the maximum dimension. For such a manifold, the sectional curvature of a plain sections containing ζ is a constant, say c .He showed that they can be divided into three classes:

(1.1) homogeneous normal contact Riemannian manifolds with $c < 0$,
(1.2) global Riemannian products of a line or a circle with a Kaehlar manifold of constant holomorphic sectional curvature if $c = 0$ and
(1.3) A warped product space $\mathfrak{R} \times_f C$ if $c > 0$.

It is well known that the manifolds of class (1.1) are characterized by admitting a Sasakian structure. Kenmotsu [8] characterized the differential geometric properties of the manifolds of class (1.3); the structure so obtained is now known as Kenmotsu structure. In general these structures are not Sasakian [8].The Gray-Hervella classication of almost Hermitian manifolds [2], there appears a class W_4 , of Hermitian manifolds which are closely related to locally conformal Kaehlar manifolds [10]. An almost contact metric structure on the manifold M is called a trans-Sasakian structure [7] if the product manifold $M \times \mathfrak{R}$ belongs to the class$W4$. The class $C_6 \oplus C_5$ (see [5], [6]) coincides with the class of trans-Sasakian structure of type (α, β) .We note that trans-Sasakian structure of type (0,0),(0, β) and (α,0) are cosymplectic [4], β -Kenmotsu [8] and α -Sasakian [8] respectively.

In 2005, Ahmet Yildiz [1] studied Lorentzian α –Sasakian manifolds and proved that conformally flat and quasi conformally flat Lorentzian α -Ssaskian manifolds are locally isometric with a sphere.

A Riemannian manifold M are locally symmetric if its curvature tensor R satisfies $\nabla R=0$, where Levi-Civita connection of the Riemannian metric. As a generalization of locally symmetric spaces, many geometers have considered semi-symmetric spaces and in turn their generalizations. A Riemannian manifold M is said to be semi-symmetric if its curvature tensor R satisfies

$$R(X,Y) \cdot R = 0, \qquad X, Y \in TM,$$

where $R(X,Y)$ acts on R as a derivation.

Locally symmetric and semi-symmetric P-Sasakian manifolds are studied in [14] .After curvature tensor, the Weyl conformal curvature tensor C and the concircular curvature tensor Z are the next important curvature tensor .In this paper, we study several derivation conditions on Lorentzian α – Sasakian manifolds. The

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paper is organized as follows. In section2, we give a brief account of Lorentzian α –Sasakian manifolds, the Wey conformal curvature tensor and the concircular curvature tensor. In section 3, we find the necessary and sufficient condition for Lorentzian α–Sasakian manifolds satisfying the condition $Z(\zeta, X)\cdot Z=0$, $Z(\zeta, X)\cdot R=0$, $R(\zeta, X)\cdot Z=0$, $Z(\zeta, X)\cdot S=0$, and $Z(\zeta, X)\cdot C=0$.

II. Lorentzian α -Sasakian Manifolds

An n- dimension differentiable manifold M is called Lorentzian α –Sasakian manifold if it admits a $(1,1)$ tensor field φ ,a contravarient vector field ζ , a covariant vector field η and a Lorentzian metric g which satisfy (see [1])

$$\eta(\zeta)=-1, \tag{2.1}$$

$$\varphi^2=I+\eta\otimes\zeta, \tag{2.2}$$

$$g(\varphi X,\varphi Y)=g(X,Y)+\eta(X)\eta(Y), \tag{2.3}$$

$$g(X,\zeta)=\eta(X), \tag{2.4}$$

$$\varphi\zeta=0, \qquad \eta(\varphi X)=0, \tag{2.5}$$

for all $X,Y \in TM$.
Also Lorentzian α -Sasakian manifold is satisfying (see [1])

$$(a)\ \nabla_X\zeta=-\alpha\varphi X, \qquad (b)\ (\nabla_X\eta)(Y)=-\alpha g(\varphi X,Y), \tag{2.6}$$

where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g Further on Lorentzian α – Sasakian manifold M the following relations holds ([1]).

$$\eta(R(X,Y)Z)=\alpha^2\{g(Y,Z)\eta(X)-g(X,Z)\eta(Y)\}, \tag{2.7}$$

$$g(R(\zeta,X)Y,\zeta)=-\alpha^2\{g(X,Y)-\eta(X)\eta(Y)\}, \tag{2.8}$$

$$(R(\zeta,X)Y=\alpha^2\{g(X,Y)\zeta-\eta(Y)X\}, \tag{2.9}$$

$$(R(X,Y)\zeta=\alpha^2\{\eta(Y)X-\eta(X)Y\}, \tag{2.10}$$

$$R(\zeta,Y)\zeta=\alpha^2\{\eta(Y)Y+Y\}, \tag{2.11}$$

$$(\nabla_X\varphi)(Y)=\alpha^2\{g(X,Y)\zeta-\eta(Y)X\}, \tag{2.12}$$

$$S(X,\zeta)=(n-1)\alpha^2\eta(X), \tag{2.13}$$

An almost para contact Riemannian manifold M is said to be η -Einstein if the Ricci operator Q satisfies

$$Q=aId+b\eta\otimes\zeta,$$

where a and b are smooth functions on the manifold. In particular if $b=0$, then M is an Einstein manifold. Let (M, g) be an n – dimensional Riemannian manifold .Then the concircular curvature tensor and the Wey conformal curvature tensor are defined by 9.

$$Z(X,Y)U=R(X,Y)U-\frac{\tau}{n(n-1)}\left[g(Y,U)X-g(X,U)Y\right], \tag{2.14}$$

$$C(X,Y)U=R(X,Y)U-\frac{1}{(n-2)}\left[S(Y,U)X-S(X,U)Y+g(Y,U)QX-g(X,U)QY\right]+\frac{\tau}{(n-1)(n-2)}\left[g(Y,U)X-g(X,U)Y\right], \tag{2.15}$$

Ref.

[1] Ahmit Yildiz and Cengizhan Murathan, On Lorentzianα–Sasakian manifolds,Kyungpook Math.J.,45,2005,pp.95-103.

[9] K.Yano and M.Kon, Structure on manifolds, Series in Pure Math, 3, Words Sci., 1984

for all $X,Y,U \in TM$, ,respectively, where R is the curvature tensor, S is the Ricci tensor and τ is the scalar curvature tensor of M .

III. MAIN RESULTS

In this section, we obtain necessary and sufficient condition for Lorentzian α –Sasakian manifolds satisfying the derivations conditions $Z(\zeta, X) . Z = 0$, $Z(\zeta, X) . R = 0$, $R(\zeta, X) . Z = 0$, $Z(\zeta, X) . S = 0$, and $Z(\zeta, X) . C = 0$.

Notes

Theorem 3.1. *An n – dimensional Lorentzian α–Sasakian manifold (M^n, g) satisfies*

$$Z(\zeta, X)\cdot Z = 0$$

if and only if either the scalar curvature of (M^n,g) is $\tau = \alpha^2\, n\,(1\text{-}n)$ or (M^n,g) is locally isometric to the Hyperbolic space $H^n(-\alpha^2)$.

Proof. In a Lorentz an α – Sasakian manifold (M^n,g) , we have

$$Z(X,Y)\zeta = \left[\left(\alpha^2 - \frac{\tau}{n(n-1)}\right)\right]\left(\eta(Y)X - \eta(X)Y\right), \tag{3.1}$$

$$Z(\zeta,X)Y = \left[\left(\alpha^2 - \frac{\tau}{n(n-1)}\right)\right]\left(g(X,Y)\zeta - \eta(Y)X\right). \tag{3.2}$$

The condition $Z(\zeta, X) . Z = 0$ implies that

$$[Z(\zeta,U), Z(X,Y)]\zeta - Z(Z(\zeta,U)X,Y)\zeta - Z(X, Z(\zeta,U)Y)\zeta = 0,$$

This in view of (3.1) and (3.2) gives

$$\left(\alpha^2 + \frac{\tau}{n(n-1)}\right)\left[Z(X,Y)U - \left(\alpha^2 - \frac{\tau}{n(n-1)}\right)\right]\{(g(Y,U)X - g(X,U)Y)\} = 0.$$

Therefore either the scalar curvature $\tau = \alpha^2 n\,(1\!-\!n)$ or

$$Z(X,Y)U = \left(\alpha^2 - \frac{\tau}{n(n-1)}\right)((g(Y,U)X - g(X,U)Y)) = 0,$$

This in view of (2.14) gives

$$R(X,Y)U = -\alpha^2(g(X,U)Y - g(Y,U)X).$$

The above equation implies that is of constant curvature $-\alpha^2$ and consequently it is locally isometric to the Hyperbolic space $H^n(-\alpha 2)$. Conversely, if has scalar curvature $\tau = \alpha^2\, n\,(1\!-\!n)$ .Then from (3.2), it follows that $Z(\zeta, X) = 0$ .Similarly in the second case, since is of constant curvature $\tau = \alpha^2 n\,(1\!=\!n)$ therefore we again get $Z(\zeta, X) = 0$ .In view of the fact $Z(\zeta, X).R$ denotes acting on R as a derivation, we state the following result as the theorem

Theorem3.2. *An n –dimensional Lorentzian α –Sasakian manifold (M^n, g) satisfies*

$$Z(\zeta, X)\cdot R = 0$$

if and only if either (M^n, g) is locally isometric to the Hyperbolic space $H^n(-\alpha^2)$.or the scalar curvature of (M^n, g) is $\tau = \alpha^2 n\,(1\!=\!n)$.

Proposition3.3. In an n –dimensional Riemannian manifold, we have $R\cdot Z = R\cdot R$

Proof. We suppose that $X, Y, U, V, W \in TM$.Therefore

$$(R(X,Y)\cdot Z(U,V,W)) = R(X,Y)Z(U,V)W - Z(R(X,Y)U,V - Z(U,R(X,Y)V)W - Z(U,V)R(X,Y)W.$$

which in view of (3.1) and symmetric properties of R ,we get

$$(R(X,Y)\cdot Z(U,V,W)) = R(X,Y)R(U,V)W - R(R(X,Y)U,V)W - R(U,R(X,Y)V)W - R(U,V)R(X,Y)W.$$

$$= (R(X,Y)\cdot R)(U,V,W).$$

This proves the proposition3.3
Now, in view of theorem2.1 12 and the proposition3.3 we have the following result as the theorem:

Theorem3.4. An $n-$ dimensional Lorentzian $\alpha-$ Sasakian manifold (M^n, g) satisfies

$$R(\zeta, X)\cdot Z = 0$$

if and only if either (M^n,g) is locally isometric to the Hyperbolic space Hn$(-\alpha^2)$.

Next we prove the following result

Theorem3.5 An $n-$ dimensional Lorentzian $\alpha-$ Sasakian manifold (M^n,g) satisfies

$$Z(\zeta, X)\cdot S = 0$$

if and only if either (M^n,g) has the curvature $\tau = \alpha^2$ n (1–n) or M^n is an Einstein manifold.

Proof. The condition $Z(\zeta, X).S = 0$ implies that

$$S(Z(\zeta, X)Y, \zeta) + S(Y, Z(\zeta, X)\zeta) = 0,$$

This in view of (2.13) and (3.2) gives

$$\left(\alpha^2 - \frac{\tau}{n(n-1)}\right)\left[S(X,Y) + \alpha^2(n-1)g(X,Y)\right]$$

Therefore either the scalar curvature of (M^n,g) is $\tau = \alpha^2 n\,(1-n)$ which is of constant or $S = \alpha^2(1-n)g(X,Y)$ which implies that (M^n, g) is an Einstein manifold with $\tau = \alpha^2 n\,(1-n)$.

which proves that theorem3.5.

Theorem3.6 *.An $n-$dimensional conformally flat Lorentzian α–Sasakian manifold (M^n,g) is locally isometric to the hyperbolic space $H^n(-\alpha^2)$.*

Proof. In this section we suppose that $Z(X,Y).U=0$ .Then from (2.14) we get

$$R(X,Y)U = \frac{\tau}{n(n-1)}\left[g(Y,U)X - g(X,U)Y\right], \tag{3.3}$$

From (3.3), we have

$$\tilde{R}(X,Y,U,W) = \frac{\tau}{n(n-1)}\left[g(Y,U)g(X,W) - g(X,U)g(Y,W)\right], \tag{3.4}$$

where $\tilde{R}(X,Y,U,W) = g(R(X,Y,U)W)$.

Putting $X=W=\zeta$ in (3.4) and by use of (2.4) and (2.8), we obtain

$$\left(\alpha^2 - \frac{\tau}{n(n-1)}\right)\left[g(Y,U) + \eta(Y)\eta(U)\right] = 0,$$

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This shows that either $\tau = \alpha^2 n(n-1)$ or $g(Y,U) = -\eta(Y)\eta(U)$ .But if $g(Y,U) = -\eta(Y)\eta(U)$ .Then from (2.3) we get $g\varphi(Y,\varphi U) = 0$, which is not possible. Therefore, $\tau = \alpha^2 n(n-1)$ .Now putting $\tau = \alpha^2 n(n-1)$ in (3.3), we find

$$R(X,Y)U = \alpha^2[g(Y,U)X - g(X,U)Y]$$

This proves the theorem3.6

$$Z(\zeta, X)\cdot C = 0$$

Notes

Theorem6. *An $n-$ dimensional Lorentzian α −Sasakian manifold (M^n, g) satisfies*

if and only if either (M^n, g) has the scalar curvature $\tau = \alpha^2 n(n-1)$ or (M^n, g) is an η -Einstein manifold.

Proof. The condition $Z(\zeta, X).C = 0$ implies that

$$[Z(\zeta,U),C(X,Y)]W - C(Z(\zeta,U)X,Y)W - C(X,Z(\zeta,U)Y)W = 0,$$

This in view of (3.1) gives

$$\left(\alpha^2 - \frac{\tau}{n(n-1)}\right)\begin{bmatrix} C(X,Y,W,U)\zeta - \eta(C(X,Y)W)U - g(U,X)C(\zeta,Y)W \\ +\eta(X)C(U,Y,W) - g(U,Y)C(X,\zeta,W) + \eta(Y)C(X,U,W)\end{bmatrix} = 0,$$

So either scalar curvature of (M^n, g) is $\tau = \alpha^2 n(n-1)$ or the equation

$$\begin{bmatrix} C(X,Y,W,U)\zeta - \eta(C(X,Y)W)U - g(U,X)C(\zeta,Y)W \\ +\eta(X)C(U,Y,W) - g(U,Y)C(X,\zeta,W) + \eta(Y)C(X,U,W)\end{bmatrix} = 0,$$

holds on M .Taking inner product of above last equations with ζ , we get

$$\begin{bmatrix} -C(X,Y,W,U)\zeta - \eta(C(X,Y)W)\eta(U) - g(U,X)\eta(C(\zeta,Y)W) \\ +\eta(X)\eta(C(U,Y,W)) - g(U,Y)\eta(C(X,\zeta,W)) + \eta(Y)\eta(C(X,U,W))\end{bmatrix} = 0,$$

Hence by using (2.7)(2.13)and(2.15) in above equations we get

$$S(X,U) = \left(\alpha^2 + \frac{\tau}{(n-1)(n-2)}\right)g(X,U) + \left(\alpha^2 + \frac{\tau}{(n-1)(n-2)} + \alpha^2(n-1)\right)\eta(X)\eta(U),$$

which implies that (M^n, g) is an η -Einstein manifold

This proves the theorem 6.

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